

# Problem 85 with PVS

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We prove that 3 divides  $n$  iff 3 divides the sum of  $n$ 's digits.

## 1 A No-Frills Encoding

```
1 p85: THEORY
2 BEGIN
3 n: VAR nat
4
5 sumdig(n): RECURSIVE nat =
6   LET d = ndiv(n,10), r = rem(10)(n)
7   IN r + IF d > 0 THEN sumdig(d) ELSE 0 ENDIF
8   MEASURE n
9
10 ten_vs_three: LEMMA
11   rem(3)(n) = rem(3)(10 * n)
12 sumdig_mod_three: LEMMA
13   rem(3)(sumdig(n)) = rem(3)(n)
14 three_divides_nat: THEOREM
15   divides(3,n) IFF divides(3,sumdig(n))
16 END p85
```

The three claims above were generated in reverse order: whenever I found a simpler claim worth proving separately, I backed out and tried that. The proofs are straightforward and uninspiring, resulting in the summary:

Proof summary for file p85.pvs

Proof summary for theory p85

sumdig_TCC1.....	proved - complete	[shostak] (0.19 s)
sumdig_TCC2.....	proved - complete	[shostak] (0.17 s)
ten_vs_three.....	proved - complete	[shostak] (0.20 s)
sumdig_mod_three.....	proved - complete	[shostak] (0.82 s)
three_divides_nat.....	proved - complete	[shostak] (0.25 s)
Theory p85 totals: 5 formulas, 5 attempted, 5 succeeded (1.63 s)		

Totals for p85.pvs: 5 proofs, 5 attempted, 5 succeeded (1.63 s)